# Fluctuation properties of strength function phenomena: A model study

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(Received 4 September 1996; revised manuscript received 20 December 1996)

We study fluctuation properties of strength function phenomena by employing a quantum mechanical model where a single parent state couples with a large number of background states. The background system is devised in such a way that the classical dynamics of the system may show a regular, an irregular, or a chaotic character as a function of a single parameter. The coupling of the parent state to the background states produces a fragmentation of the parent state, giving rise to a strength function phenomenon. We study various measures of the strength function that characterize its bulk structure or fluctuation properties. They include energy moments, strength distribution, fractal dimensions of the strength function, and Fourier transform of the autocorrelation function. Some of these measures, such as strength distribution or Fourier transform of the autocorrelation function, reflect characteristic aspects of the dynamics of the background system, i.e., if they have a regular or a chaotic character, while measures such as energy moments or fractal dimensions are rather insensitive to the dynamics. [S1063-651X(97)01406-2]

PACS number(s): 05.45.+b, 05.40.+j, 03.65.Sq

## I. INTRODUCTION

In finite quantum mechanical many-body systems such as nuclei, hadrons, or clusters, detailed spectroscopic studies have been carried out from which much information on the dynamics of the complex quantum system can be extracted. Sometimes the energy spectrum alone provides a clue to the dynamical nature of the system: Rotational spectrum, e.g., is a typical example for such a case. It is more common, however, that a detailed knowledge can be obtained from the response of the system to an external field which couples to a specific degree of freedom of the system. We hereafter call the strength of a response as a function of energy the strength function.

In many cases an emphasis is placed on the bulk (or gross) properties of the strength function, e.g., the peak position and its strength, or the width of the main peak, etc. Many of these quantities are related to low order (energy) moments of the strength function, and are often constrained by a sum rule. Our main interest in this paper is, however, concerned more with the fluctuation properties of the strength function. One of the quantities which reflects such a fluctuation property is the strength distribution. Already in the 1950s a study of the neutron strength function in nuclei at low energies revealed that the strength distribution shows a significant statistical feature, the Porter-Thomas-type distribution, which is obtained from the random matrix theory [1,2]. Recently the fluctuation properties of energy spectra and strength functions have been studied from the viewpoint of ''quantum chaos,'' for instance, see Ref. [3]. In contrast to an average structure of the strength function which would be specific to the detailed dynamics of the system, at least some of the fluctuation properties are believed to be universal, i.e., characteristic to a class of many complex systems [4]. The strength distribution, for instance, is one of many quantities which reflect fluctuation properties of the strength function. In particular, it contains no information on the energy-strength correlation. The latter may be reflected in other quantities constructed from the strength function, e.g., the autocorrelation function of the strength.

The purpose of the present paper is to study various fluctuation properties of the strength function of a model quantum system and to investigate possible signatures which may reflect an underlying dynamical character of the system. The model system is devised so as to generate a strength function which would cover different dynamical structures. More specifically, the strength function phenomenon in our model system arises as a result of the coupling of a single parent state with a large number of background states, the latter being classically integrable or chaotic depending on the value of a single parameter. For this system we calculate several quantities which characterize the structure of the strength function. The present paper is an extension of the study in our previous paper [5] where a slightly different model has been used. The choice of the coupling Hamiltonian in this paper, in particular, would be more suitable because the sum and the width of the strength are conserved for different parameters.

The paper is organized as follows. In Sec. II the model is presented. We consider also a model based on the random matrix which is used as a reference of a fully chaotic system. It is another purpose of the present paper to clarify whether we can see the differences between the dynamically chaotic case and the case of the random matrix model in the fluctuation of the strength function. Strength function of the system is calculated and analyzed in Sec. III. Energy-weighted mo-

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ments and the strength distribution are studied in detail. In Sec. IV we perform a moment analysis based on a partition function similar to the one developed for a multifractal system. This may serve as one possible measure of the energy-strength correlation. The autocorrelation function of the strength function is also an interesting quantity which reflects the fluctuation properties of the strength function. We calculate the Fourier transform of the autocorrelation function in Sec. V and compare with previous studies made in other systems [6]. The final section is devoted to a brief summary.

## II. MODEL

### A. Formulation of the model

The model space is composed of a single state, which we call a parent state from now on, and some large number of background states,

$$\{|c\rangle,|i\rangle;i=1,\ldots,N_{bg}\}. \tag{2.1}$$

The state $|c\rangle$ represents the parent state with an unperturbed energy $\epsilon$, the strength function of which is a main focus of this paper. The states $|i\rangle$ represent the background states. The total Hamiltonian is given by

$$H=H_c+H_{bg}+V_{coupl}. \tag{2.2}$$

Here, $H_c$ acts only on the state $|c\rangle$ and is written as

$$H_c=\epsilon|c\rangle\langle c|, \tag{2.3}$$

$H_{bg}$ acts on the background states $|i\rangle$, and $V_{coupl}$ represents a coupling between the state $|c\rangle$ and the states $|i\rangle$. We adopt as the Hamiltonian $H_{bg}$ of the background system a coupled two-dimensional anharmonic oscillator characterized by a single parameter $k$,

$$H_{bg}=a(k)H_{anh}, \tag{2.4a}$$

$$H_{anh}=\frac{1}{2}(p_x^2+p_y^2+x^4+y^4)-kx^2y^2. \tag{2.4b}$$

As the value of the parameter $k$ increases, the classical phase space structure of the Hamiltonian $H_{anh}$ changes from regular to almost completely chaotic characters [7]. The states $|i\rangle$ are eigenfunctions of $H_{anh}$ and hence of $H_{bg}$. The energy of the state $|i\rangle$ associated with $H_{bg}$ is denoted by $\omega_i$ whose value is scaled by $a(k)$ from the original eigenvalue $\omega_i^{ORG}$ of $H_{anh}$. We introduce the parameter $a(k)$ in order that the mean level density of the background system remains the same for various $k$. As basis states for the diagonalization of $H_{anh}$, we took eigenstates of an uncoupled two-dimensional harmonic oscillator with frequency $\Omega(=\Omega_x=\Omega_y)$. They are denoted by $|\mu\rangle$, where $\mu$ stands for a pair of integers, i.e., numbers of oscillator quanta in the $x$ and $y$ directions. The value of $\Omega$ was determined for each $k$ so as to optimize the diagonalization [8,5].

The strength function of the parent state depends on the choice of $V_{coupl}$. Since our purpose is to study the effect of the background system dynamics on the strength function, $V_{coupl}$ must be simple enough to make an analysis. One possible choice adopted in this paper is

$$V_{coupl}=\chi(k)\sum_{\mu}(|c\rangle\langle\mu|+\text{H.c.}). \tag{2.5}$$

This means that the state $|c\rangle$ couples with every basis state of the background with an equal strength $\chi(k)$. The actual coupling matrix elements of the state $|c\rangle$ to the eigenstates of the background system are then given by

$$v_i\equiv\langle c|V_{coupl}|i\rangle=\chi(k)\sum_{\mu}\langle\mu|i\rangle. \tag{2.6}$$

These matrix elements reflect the complexity of the amplitudes of the states $|i\rangle$. It will be shown later that $v_i$ show a random behavior when the background system becomes chaotic. Note that the present choice of the coupling is slightly different from that adopted in our previous paper [5].

The classical or quantal features of $H_{anh}$ have been studied in detail; for instance, see Refs. [9–13]. The procedure of diagonalization of $H_{anh}$ is also explained there. Thus we describe here only the actual values of parameters adopted in the numerical calculation.

We adopted three values of the parameter $k$ in $H_{anh}$, i.e., $k=0.0$, 0.2, and 0.6. They are considered as typical values for an integrable, partially irregular, and almost chaotic systems, respectively. As for the background states $|i\rangle$ we consider only those eigenstates of $H_{anh}$ which belong to one symmetry class, i.e., that symmetric in the $x$, $y$, and diagonal directions. $H_{anh}$ is diagonalized within a large space which is composed of 5776 basis states $|\mu\rangle$ having the same symmetry. The values of frequency $\Omega$ of the uncoupled two-dimensional harmonic oscillator, determined so that the trace of $H_{anh}$ in this space is minimized, are 8.4438, 7.5, and 7.15 for $k=0.0$, 0.2, and 0.6, respectively [5]. We then pick up the lowest 800 $(=N_{bg})$ eigenstates of $H_{anh}$ as the background states $|i\rangle$. The parameter $a(k)$ is adjusted so that the energies of the background states are scaled; $a(k)=400/\omega_{800}^{\text{orig.}}$. This makes the mean level densities of the background states for different $k$ values to be constant, i.e., $\overline{\rho}=2.0$.

The value of $\epsilon$ is fixed to 200, so that the state $|c\rangle$ is located in the middle of the background 800 states, and thus a large number of background states can be found in the neighborhood.

The coupling strength $\chi(k)$ is determined so that the sum of $v_i^2$ is independent of $k$, and is fixed by the condition $\Sigma_{i=1}^{800}v_i^2=800$. This implies that the average coupling strength of the state $|c\rangle$ to the background states remains constant, which allows us to make a fair comparison of the results for different $k$ values. The resulting values of $\chi(k)$ are 0.670, 0.714, and 0.638 for $k=0.0$, 0.2, and 0.6, respectively. One may notice that $\chi(k)$ would be constant if the number of the basis states $|\mu\rangle$ is kept equal to that of the adopted states $|i\rangle$. We included many more states in the former since at least for large $k$ a mixing of the basis states should be important to fully retain the fluctuation properties of the eigenstates.

The strength function of the parent state $|c\rangle$ is then defined by

$$S(E) \equiv \sum_n \delta(E - E_n) S_n , \tag{2.7a}$$

$$S_n \equiv |\langle c|n\rangle|^2, \tag{2.7b}$$

where $|n\rangle$ denotes an eigenstate of the total system and $E_n$ the corresponding eigenvalue.

### B. Random matrix model

We consider another model for comparison, which we call the random matrix model. It is essentially the same as that used in Ref. [14]. We consider an ensemble of systems, each of which is similar to the one presented above except for the choice of the background system. Here the background states $|i\rangle$ are obtained by diagonalizing each realization of the random matrices which obey the Gaussian orthogonal ensemble (GOE) [2]. The eigenvalues of the random matrix are distributed according to the semicircle law between $-N/2$ and $N/2$ where $N$ denotes the dimension of the matrix, and the mean level density $\overline{\rho}$ unity. We fix $N = N_{\rm bg} = 800$. The energy of the parent state $|c\rangle$ is fixed to zero so that it is always located just in the middle of the background states $|i\rangle$. As for the coupling between the state $|c\rangle$ and the background states, we take the same form as Eq. (2.5), and replace the states $|\mu\rangle$ with the basis states for the diagonalization of the random matrix. The coupling strength is fixed at $\chi = 2.0$ so that $\overline{\rho^2} \Sigma v_i^2 = 3200$ and is equal to that of the model in the preceding subsection. The strength function is again defined in the same manner.

## III. STRENGTH DISTRIBUTION

### A. Distribution of coupling matrix elements

We first consider the distribution of the coupling matrix elements $v_i$. Figure 1 shows the distribution of $v_i$ for $k = 0.0$, 0.2, and 0.6 and for the random matrix model. The matrix element values for $k = 0.0$ show a concentration at around $\pm 1$. This result is understood from the presence of dominant terms among the coefficients $\langle \mu|i\rangle$ in the sum of Eq. (2.6) for $v_i$. On the contrary, the distribution for $k = 0.6$ is almost a Gaussian centered at zero and with width 1.0. An inspection of the expression (2.6) suggests that for $k = 0.6$ the values of $\langle \mu|i\rangle$ are independently random and there are no dominant terms in the sum in accordance with the central limit theorem. However, this does not imply that the distribution of $\langle \mu|i\rangle$ itself, namely, the amplitude distribution of the background states, should follow a Gaussian. Indeed the amplitude distribution for $k = 0.6$ has an additional peak around zero over the Gaussian-like distribution. This is different from the random matrix model where the distribution of $\langle \mu|i\rangle$ as well as that of $v_i$ follow a Gaussian.

### B. Gross structure of the strength function

Figure 2 shows the strength function $S(E)$. In spite of the difference in the distribution of the coupling matrix elements as seen above, the shapes of $S(E)$ look rather similar to each other. For the sake of a quantitative discussion on the gross structure of the strength function, let us consider the $n$th energy central moment [15] of the strength function $\mathcal{M}_E^{(n)}$, which is defined as

$$\mathcal{M}_E^{(n)} \equiv \langle\!\langle (E - \langle E\rangle)^n \rangle\!\rangle, \tag{3.1}$$

where $\langle E^n\rangle$ is also defined as

$$\langle E^n\rangle \equiv \int E^n S(E) dE. \tag{3.2}$$

Table I lists the calculated results for $\mathcal{M}_E^{(n)}$ for $n = 3$ to 10 at the three chosen $k$ values. It includes also those for a schematic Lorentzian-type strength function for a comparison. It is defined as

$$S(E_n) \equiv \frac{\mathcal{N}}{(E_n - \epsilon)^2 + (\Gamma/2)^2},$$

$$E_n = \frac{1}{2}(n-1) \quad (n = 1, \ldots, 801), \quad \epsilon = 200 \tag{3.3}$$

where $\Gamma$ is fixed so that $\mathcal{M}_E^{(2)} = 800$, and $\mathcal{N}$ is a normalization constant. Because of the sum rule [16]

$$\langle\!\langle (E - \langle E\rangle)^n \rangle\!\rangle = \langle c|(H - \epsilon)^n|c\rangle, \tag{3.4}$$

the values of the average and the variance are constrained as

$$E_{\rm ave} \equiv \langle E\rangle = \epsilon, \quad \sigma_E^2 \equiv \mathcal{M}_E^{(2)} = \sum_i v_i^2 \tag{3.5}$$

and are independent of the parameter $k$. From Table I, we can see that the values of $\mathcal{M}_E^{(n)}$ up to $n = 10$ are similar to each other for all three $k$ values. We also see that these values are similar to those for a schematic Lorentzian-type strength function, although we find a small deviation in the odd and the tenth energy moments.

Since the moment $\mathcal{M}_E^{(n)}$ can be generally written as

$$\mathcal{M}_E^{(n)} = f_n(\{\mathcal{M}_E^{(m)},\ m < n\}) + \sum_i v_i^2 (\omega_i - \epsilon)^{n-2}, \tag{3.6}$$

where the first term represents a polynomial function of $\mathcal{M}_E^{(m)}$'s with $m < n$, we can explain the above similarity of $\mathcal{M}_E^{(m)}$ by showing that the second term of Eq. (3.6) is almost independent of the parameter $k$. Equation (3.6) can be easily verified if we rewrite $\mathcal{M}_E^{(n)}$ as

$$\mathcal{M}_E^{(n)} = \sum_{i,i'} \langle c|H - \epsilon|i\rangle \langle i|(H - \epsilon)^{n-2}|i'\rangle \langle i'|H - \epsilon|c\rangle, \tag{3.7}$$

and decompose $\mathcal{M}_E^{(n)}$ by inserting $|c\rangle\langle c|$ or $\Sigma_j |j\rangle\langle j|$ between $H - \epsilon$ factors; if we insert $|c\rangle\langle c|$ more than once, we obtain products of $\mathcal{M}_E^{(m)}$ $(m < n)$. On the other hand, if we insert $\Sigma_j |j\rangle\langle j|$ into all places, this leads to the second term of Eq. (3.6). This term is actually insensitive to the difference of the value of $k$. This is because the average value of $v_i^2$ as a function of energy has a similar shape independent of $k$, as

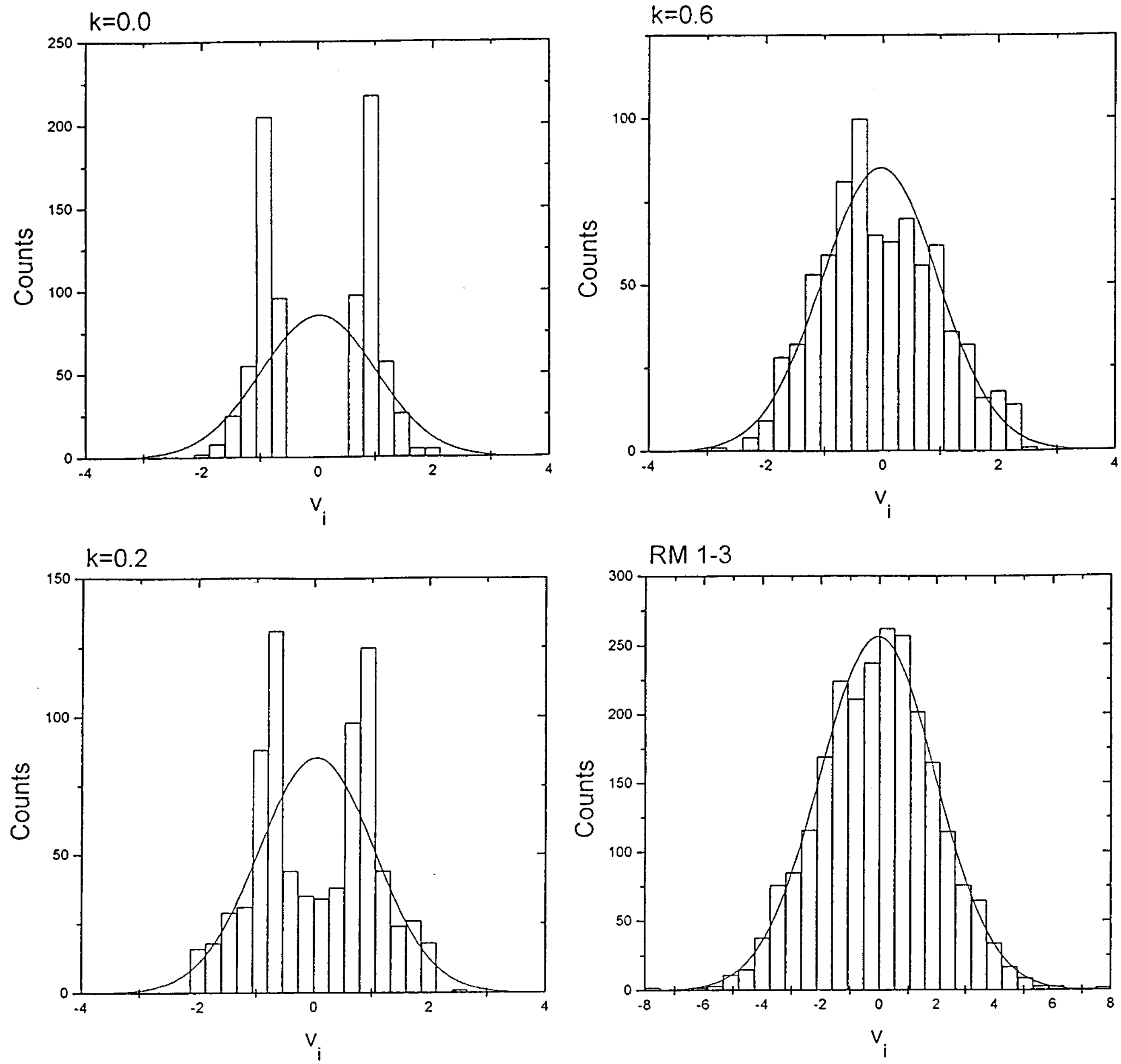


FIG. 1. Distribution of the coupling matrix elements $v_i$ for three values of $k$ and also for the random matrix model. For the latter case, the result for three choices of realization is presented. The smooth curves show a normalized Gaussian distribution having the same width as that of the $v_i$ distribution for each case.

can be seen in Fig. 3, which shows $\omega$ dependence of the averaged value of $v_i^2$ defined as

$$v_{\rm ave}^2(\omega)\equiv\frac{\sum_{\omega-(1/2)\delta\omega\leqslant\omega_i\leqslant\omega+(1/2)\delta\omega} v_i^2}{\sum_{\omega-(1/2)\delta\omega\leqslant\omega_i\leqslant\omega+(1/2)\delta\omega} 1}. \quad (3.8)$$

Here we took $\delta\omega=20$. Moreover, this is also because the local level density of the background states is independent of $k$. Thus we do not find significant difference in the gross structure of the strength function as characterized by the low energy moments.

### C. Strength distribution

The apparent similarity of the strength function for different $k$ values seen in the preceding subsection is only superficial. To see this let us turn our attention to the strength distribution. The distribution is known to take the Porter-Thomas (PT) form for a pure random matrix, i.e., without coupling considered in this paper. In Fig. 4 we plot the strength distribution for three $k$ values together with the PT shape. Although not very clear, a tendency towards the PT distribution can be seen as $k$ increases. A much clearer difference can be seen if we plot the distribution of the renormalized amplitude $\sqrt{\tilde{S}_n^{(u)}}$, where

$$\tilde{S}_n^{(u)}\equiv S_n\{(E_n-\epsilon)^2+(\Gamma/2)^2\} \quad (3.9)$$

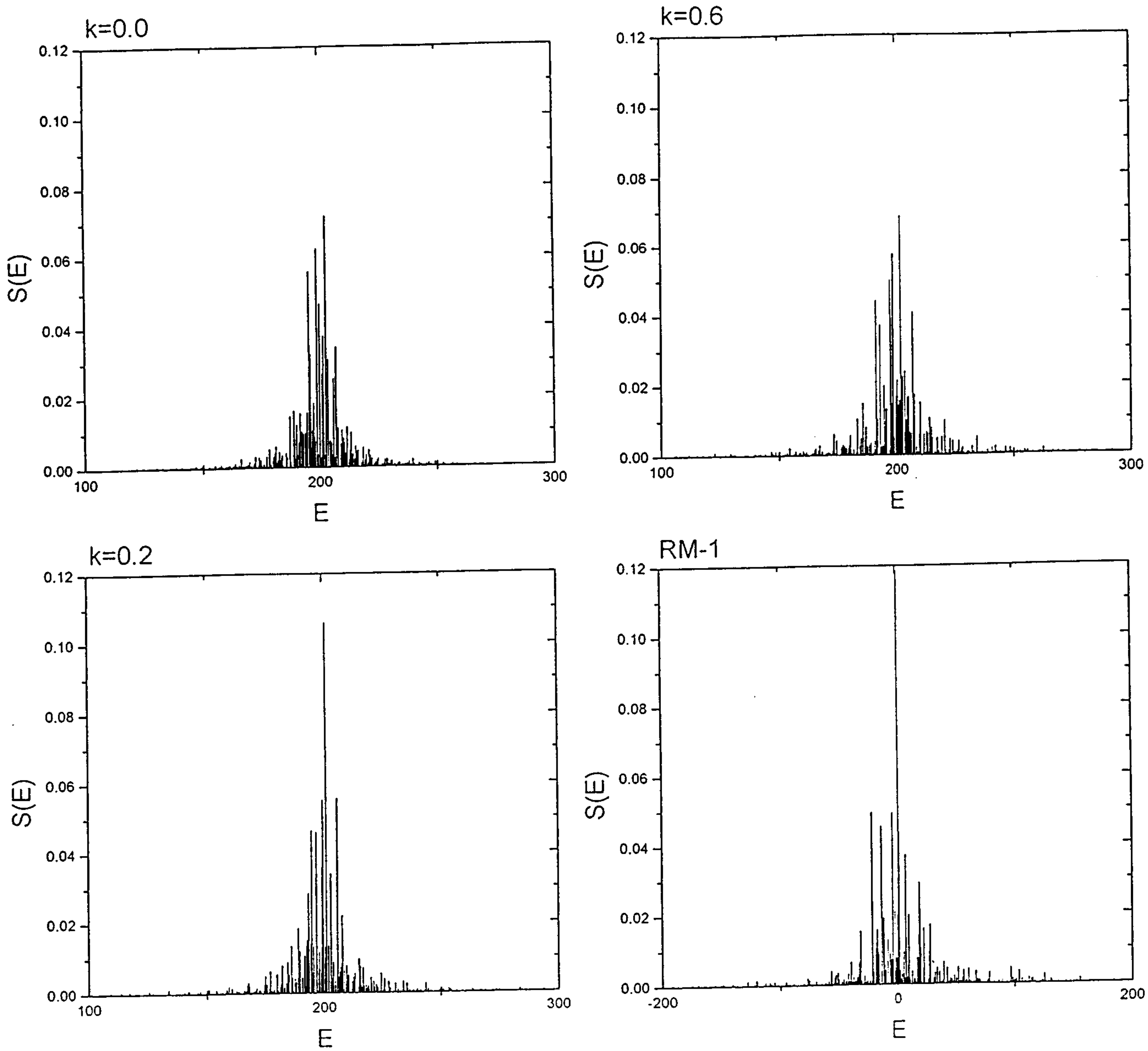


FIG. 2. Strength function $S(E)$ for the three values of $k$ and also for the random matrix model. For the latter case, the result for one choice of realization is presented.

is the strength corrected for the energy-dependent factor. This factor has been introduced to remove approximately the energy denominator contribution: if we assume constant coupling matrix elements $v_i = v_c$ and an equal level distance $D_c$ in our model, the strength function will be given by $S(E) \simeq (\Gamma/2\pi)/\{(E-\epsilon)^2 + (\Gamma/2)^2\}$, where $\Gamma = 2\pi v_c^2/D_c$ [16]. This may be contrasted to the present calculation in which the mean square value of $v_i$ gives $\overline{v_i^2} = 1$ and the mean level distance is $D \approx 0.5$ for our model of the oscillator background system, and $\overline{v_i^2} = 4$, $D \approx 1.0$ for the random matrix model. We thus choose $\Gamma = 4\pi$ and $8\pi$, respectively, for these models. The results are given in Fig. 5. The smooth curve shows a Gaussian distribution which would show up in chaotic systems as represented by the random matrix theory

TABLE I. The $n$th energy moment of the strength function $\mathcal{M}_E^{(n)}$ divided by $\sigma_E^n$.

| $k$ | 3rd | 4th | 5th | 6th | 7th | 8th (units of $10^4$) | 9th | 10th (units of $10^5$) |
|---|---|---|---|---|---|---|---|---|
| 0.0 | −0.068 | 17.889 | −2.840 | 547.10 | −124.78 | 1.9843 | −5664.4 | 7.8227 |
| 0.2 | −0.049 | 17.925 | −1.917 | 549.49 | −79.95 | 1.9980 | −3463.0 | 7.8967 |
| 0.6 | −0.048 | 17.851 | −1.986 | 545.00 | −86.73 | 1.9738 | −3913.3 | 7.7709 |
| Lorentzian | 0.0 | 17.530 | 0.0 | 527.86 | 0.0 | 1.8904 | 0.0 | 7.3709 |

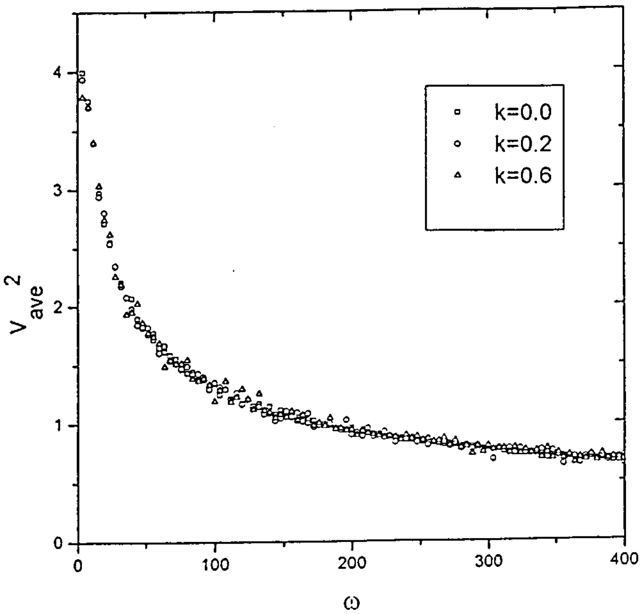


FIG. 3. $v_i^2$ averaged over an energy range of $\delta\omega = 20$ as a function of energy for $k = 0.0$, 0.2, and 0.6.

[1,2], although the actual calculation still shows a small deviation. The distributions for $k = 0.0$ and 0.2 are far from the Gaussian, while that for $k = 0.6$ is much closer. A comparison with Fig. 1 shows that the correction factor in Eq. (3.9) removes approximately the energy-denominator effect, leading back to the coupling strength distribution which directly depends on the dynamics of the background system. It would be interesting to extend this idea to a realistic system, although one has first to find how one can optimize parameters such as $\Gamma$.

In the actual application to realistic systems, one must consider also the finiteness of the energy resolution, which we have neglected above. Can the strength distribution still be a signature of the background dynamics even when one cannot resolve an individual peak? In order to partly answer this question let us consider the following distributions: define $x_n \equiv \sqrt{\widetilde{S}_n^{(u)}}$, then consider the distribution of the following quantity:

$$y_i^{(N)} = \sum_{n=N(i-1)+1}^{Ni} x_n \quad \left( i = 1,2, \ldots, \left[ \frac{800}{N} \right] \right), \tag{3.10}$$

namely, the distribution of the sum of neighboring $N$ strengths. Here, $[j]$ stands for the largest integer which is not larger than $j$. To illustrate the effect of summation, let us first assume that the distribution of $x_n$ follows a Gaussian,

$$P^{(1)}(x) = \sqrt{\frac{2}{\pi}} \frac{1}{\sigma} e^{-x^2/2\sigma^2} \quad \left( \int_0^\infty P^{(1)}(x) dx = 1 \right), \tag{3.11}$$

and that there is no correlation among $x_n$'s; then the distribution of $y_i^{(2)}$ can be calculated as

$$P^{(2)}(y) = \int_0^\infty \int_0^\infty P^{(1)}(x_1) P^{(1)}(x_2) \delta(y - x_1 - x_2) dx_1 dx_2$$
$$= \frac{4}{\pi\sigma} e^{-y^2/4\sigma^2} \left[ \frac{\sqrt{\pi}}{2} - \mathrm{erfc}\left( \frac{y}{2\sigma} \right) \right], \tag{3.12}$$

where $\mathrm{erfc}(x)$ denotes the error function,

$$\mathrm{erfc}(x) \equiv \int_x^\infty e^{-t^2} dt. \tag{3.13}$$

The distribution of $y_i^{(3)}$, $P^{(3)}(y)$, is obtained in a similar manner. The result (3.12) and that of $P^{(3)}(y)$ show that, as one may naively anticipate, the peak of $P^{(2)}(y)$ or $P^{(3)}(y)$ is shifted to a larger value even if the peak of $P^{(1)}(x)$ is located at $x = 0$. The same tendency also appears in Fig. 6 where the distributions of $y_i^{(N)}$ $(N = 2,3)$ are shown for our model with $k = 0.0$, 0.2, and 0.6. Although there still remains a difference, it is much more difficult to qualitatively distinguish the graphs for different $k$'s as compared to the case of Fig. 5: One may not be able to determine whether the deviation of the peak from zero is due to the properties of the background system or poor resolution. In the case of the empirical data the situation is more complex, because the summation of the strengths is made over a given energy interval and not with a fixed number of levels. We suspect that the strength distribution alone would not give a clear signature of the dynam-

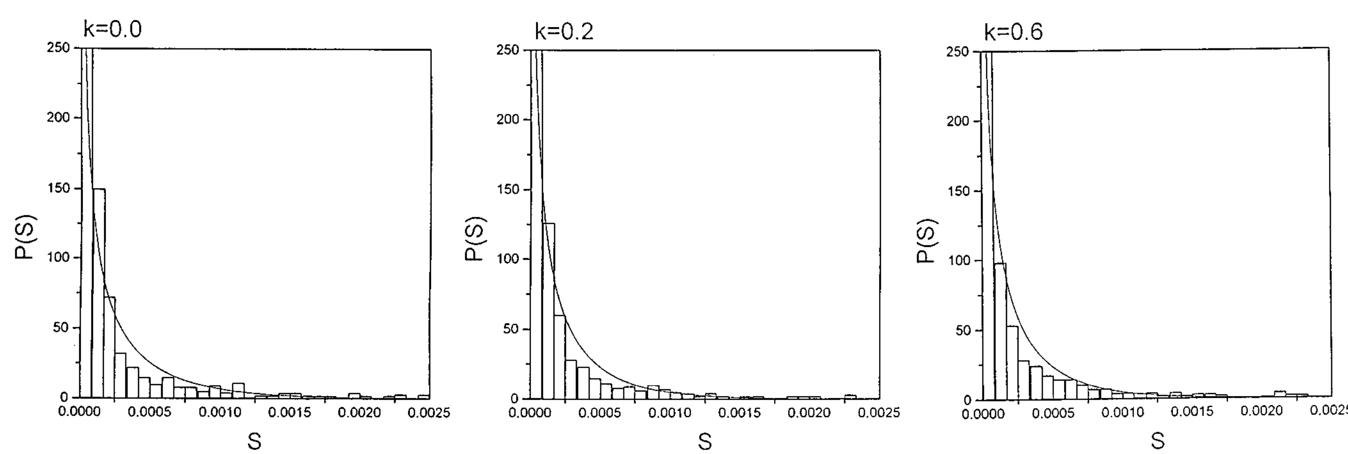


FIG. 4. Strength distribution for the three values of $k$. The smooth curves show a Porter-Thomas distribution.

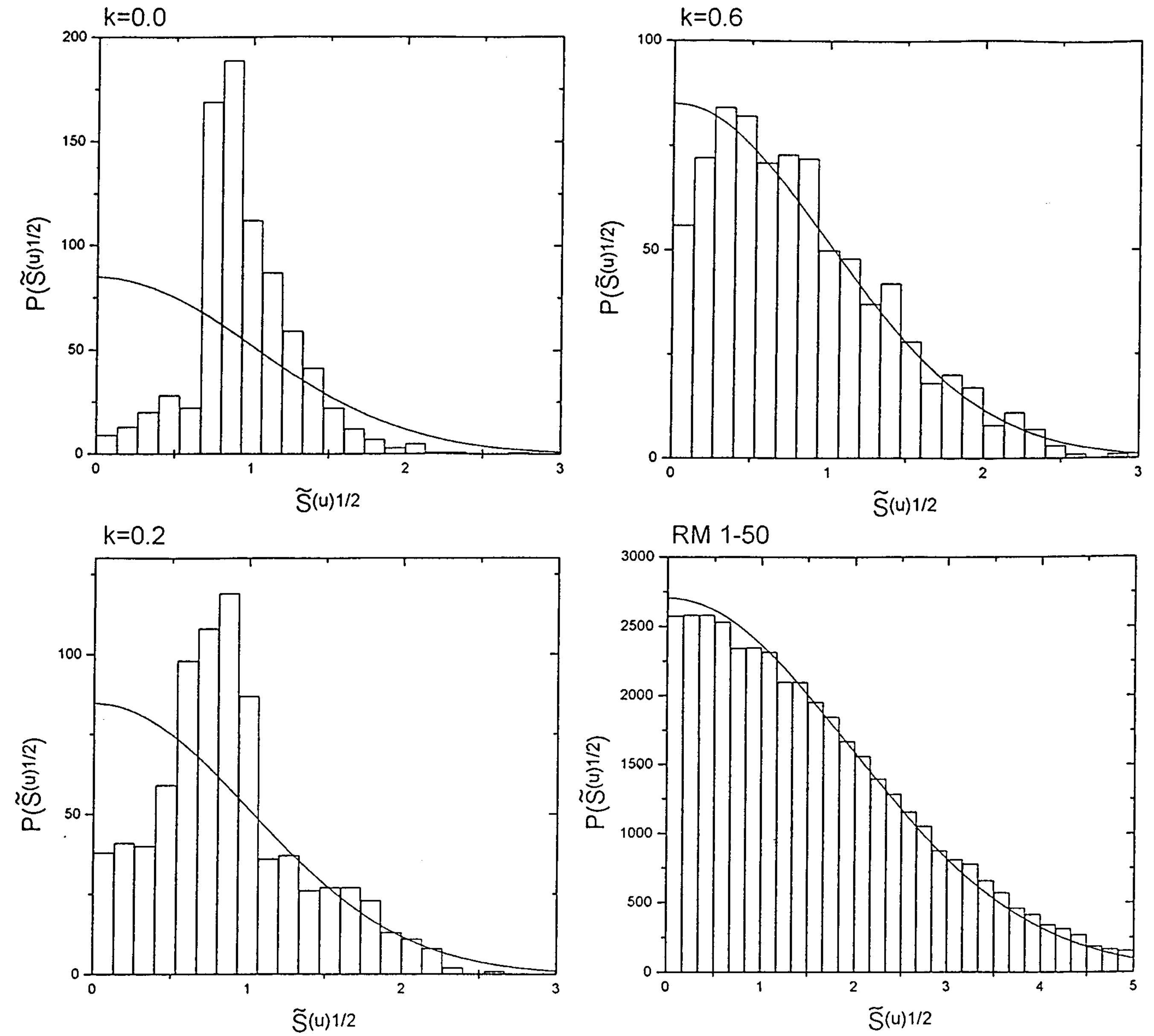


FIG. 5. Distribution $P(\sqrt{\widetilde{S}^{(u)}})$ where $\widetilde{S}^{(u)}=S((E-\epsilon)^2+(\Gamma/2)^2)$. $\Gamma$ is fixed to $4\pi$ for $k=0.0$, 0.2, and 0.6 and $8\pi$ for the random matrix model. The smooth curves show a Gaussian distribution. For the random matrix model 50 choices of realizations have been accumulated.

ics of the background system as far as the resolution is not enough to detect individual strengths.

## IV. FRACTAL ANALYSIS OF THE STRENGTH FUNCTION

The strength distribution is one of the signatures which characterize fluctuations of the strength function. It reflects only a part of the structure in the strength function of Fig. 2: For instance, it does not contain a correlation between the energy and the strength.

In this section we perform a moment analysis of the strength function similar to the one applied for a multifractal system [17,18]. This analysis takes into account some features of the energy-strength correlation, and therefore can be another characteristic measure of the strength function independent of the distribution $P(\sqrt{\widetilde{S}^{(u)}})$. Instead of using directly the energy eigenvalue $E_n$ and the strength $S_n$ we consider the following modified strength function:

$$S^{(f)}(E)\equiv\sum_n \delta(E-\widetilde{E}_n)\widetilde{S}_n, \tag{4.1}$$

where $\widetilde{E}_n$ denotes the unfolded energy and $\widetilde{S}_n$ the normalized strength in Eq. (3.9) with an additional normalization condition $\Sigma_n\widetilde{S}_n=1$. Let us now divide the whole energy interval $\Delta E(=\widetilde{E}_{801\text{th}}-\widetilde{E}_{\text{g.s.}})$ into $L$ segments each having a width $\delta E=\Delta E/L$, where $\widetilde{E}_{\text{g.s.}}$ represents the energy of the ground state. We then sum up the strengths within each segment, giving

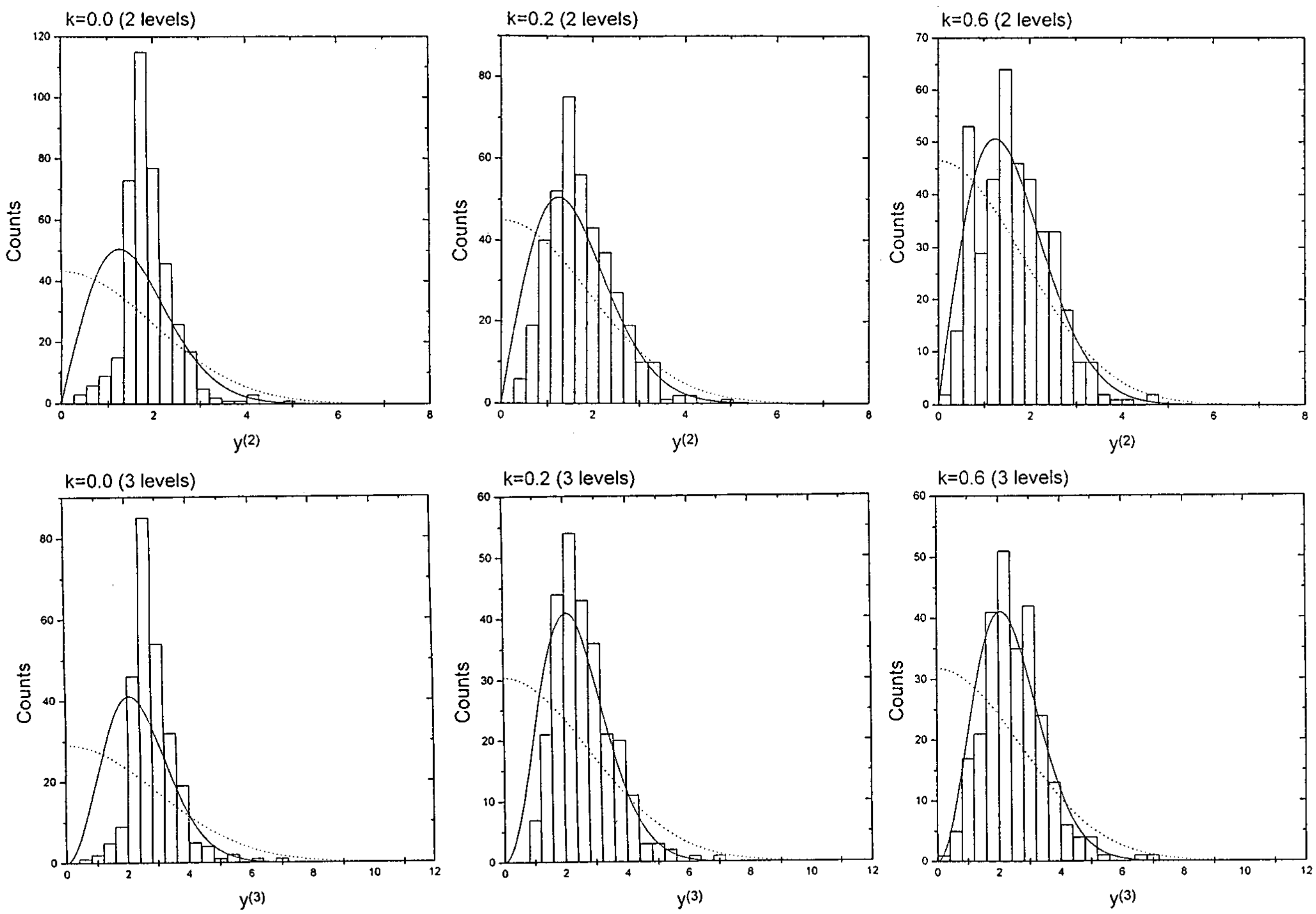


FIG. 6. Distribution of $y_i^{(N)}$ $(N=2,3)$ for three values of $k$. The lines correspond to $P^{(2)}(y)$ for $y_i^{(2)}$ and $P^{(3)}(y)$ for $y_i^{(3)}$ and the dotted curves a Gaussian. See text for more detail.

$$P_j \equiv \sum_{(j\text{th segment})} \tilde{S}_n \quad (j=1,\ldots,L), \tag{4.2}$$

with the normalization condition $\Sigma_j P_j = 1$. The partition function is defined by

$$\chi_m(\delta E) \equiv \sum_{j=1}^{L} P_j^m. \tag{4.3}$$

We then study its behavior as we refine the scale, e.g., as $L=2\rightarrow 2^2\rightarrow 2^3$, etc.

The reason we consider the modified strength function (4.1) instead of the original one follows. One of the purposes of considering the partition function (4.3) is to find whether a scaling law does hold as $\delta E$ is refined or whether there are some typical energy lengths which lead to breaking of this scaling law. However, there are two typical energy lengths in the original strength function, namely, the mean level distance and the width of the strength function $\Gamma$, both of which are out of our interest. The scaling property therefore must be considered at larger energy resolution than the mean level distance. Besides considering the energy resolution smaller than $\Gamma$, it is another method to use the normalized strength $\tilde{S}_n$ in order to avoid the global energy variation of the strength function. We chose the latter method in this paper because the available energy interval becomes wider than the case of the former method. Moreover, the scaling property of the partition function depends on how ''the support'' (the energy levels in the present case) is distributed as well as how the strengths are distributed on this support. Since we would like to focus on the latter, we used the unfolded energy in the definition of the modified strength function (4.1) in order to make the support distribute uniformly. Accordingly the analysis adopted here can be seen as an improved version of that in Ref. [5].

Figure 7 shows the dependence of the partition function $\chi_m$ for $m=2$ to 5 on the scale $\delta E$ for $k=0.0$, 0.2, and 0.6. We also show the one for the random matrix model averaged over 50 choices of the realizations of the ensemble. The results show a linear dependence in the logarithmic scale within a wide range of $\delta E$, suggesting a scaling property of $S^{(f)}(E)$. For small $\delta E$ the partition function $\chi_m$ eventually reaches a fixed value which is the consequence of the discrete spectrum in our model.

Let us consider the fractal dimension $D_m$ defined by

$$D_m \equiv \lim_{\delta E\rightarrow 0} \frac{B_m(\delta E)}{m-1}, \quad B_m(\delta E) = \frac{\ln\chi_m}{\ln\delta E}. \tag{4.4}$$

In practice, the expression for $B_m(\delta E)$ has been replaced with the ratio of the difference of $\ln\chi_m(\delta E)$ to that of $\ln\delta E$ in

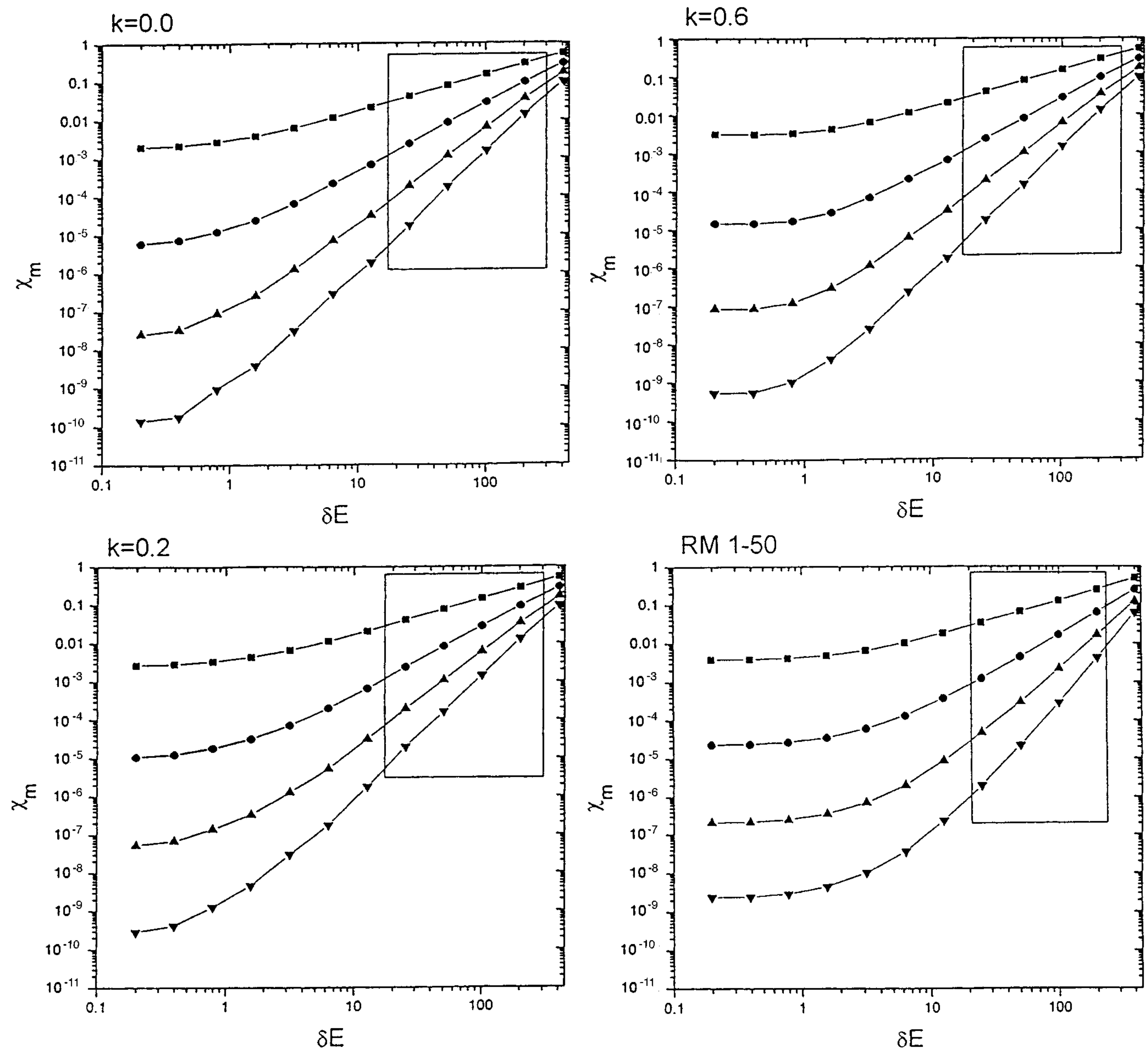


FIG. 7. $\chi_m(\delta E)$ versus $\delta E$ for three values of $k$ as well as for the random matrix model. For the latter case the result of averaging over 50 choices of the realization is presented. The lines correspond to $m=2$ to 5 from the upper to the lower ones.

the appropriate interval of $\delta E$. The quantity $D_m$ reflects a state-to-state fluctuation of the strength function. In Table II we show $D_m$ for $m=2$ to 5 calculated for each line shown in Fig. 7. The interval of $\delta E$ where $D_m$'s are calculated is indicated by a box in Fig. 7. From Table II, we find that the relation $D_m>0.9$ holds for all $m$ in the case of the random matrix model, while for the case of $k=0.0$, 0.2, and 0.6, the value of $D_m$ decreases as $m$ increases, suggesting a multifractal nature for these systems.

TABLE II. Fractal dimension $D_m$ ($m=2$ to 5) at $k=0.0$, 0.2, and 0.6 and those for the random matrix model (RMM).

| $k$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ |
|---|---|---|---|---|
| 0.0 | 0.94 | 0.89 | 0.84 | 0.80 |
| 0.2 | 0.94 | 0.88 | 0.82 | 0.78 |
| 0.6 | 0.94 | 0.89 | 0.83 | 0.79 |
| RMM | 0.97 | 0.95 | 0.93 | 0.92 |

As mentioned in Sec. III C, the modified strength function $S^{(f)}(E)$ can be thought to reflect directly the $i$ dependence of $v_i^2$. Thus we also obtained the partition function $\chi_m(\delta i)$ for $v_i^2$ as a function of $\delta i$ instead of $\delta\omega$ in the same procedure as the case of $S^{(f)}(E)$, and also obtained $D_m$, which is derived from $\chi_m(\delta i)$. We found that the results are very similar to the case of $S^{(f)}(E)$. This can be understood from Fig. 3. As known from Fig. 3, for the case of our model with an anharmonic oscillator background, $v_{\rm ave}^2$ shows the same $\omega$ dependence independent of the value of $k$. For the case of the random matrix model, however, there is no such dependence of $v_i^2$ on $\omega$; the averaged value of $v_i^2$ is independent of $\omega$. This difference of $\omega$ dependence (or $i$ dependence) of $v_i^2$ between our model and the random matrix model is reflected in the difference of the behavior of $D_m$ between them. Thus it may not be adequate to use the word multifractal for our model, because the behavior of $D_m$ in our model only reflects the smooth energy variation of $v_i^2$.

The studies in this section suggest that it is difficult to find out the difference of dynamics of background system in the behavior of the partition function $\chi_m(\delta E)$ for $m \geqslant 2$ at $\delta E$, the value of which is larger than the mean level distance, because the energy dependence of the averaged $v_i^2$ does not depend on the value of $k$. Nevertheless, the studies suggest that we can find the difference between the dynamically chaotic case and the case of the random matrix model in the fluctuation of the strength function by means of the analysis in this section.

## V. FOURIER ANALYSIS

In this section we study the Fourier transform of the strength function which provides another measure of the energy-strength correlation of the dynamical system [19]. Moreover, this quantity is expected to be insensitive to the experimental resolution of the energy spectrum [6].

We define the Fourier transform of the modified strength function defined in the preceding section as

$$C(t) = \int dE \; e^{-iEt} S^{(f)}(E). \tag{5.1}$$

In the following, we consider the square of $C(t)$; it is given by

$$|C(t)|^2 = \sum_n \tilde{S}_n^2 + 2 \sum_{n>n'} \tilde{S}_n \tilde{S}_{n'} \cos(\tilde{E}_n - \tilde{E}_{n'})t$$
$$= \int d\omega \; e^{-i\omega t} A(\omega), \tag{5.2}$$

where

$$A(\omega) = \int dE \; S^{(f)}(\omega + E) S^{(f)}(E) \tag{5.3}$$

is the autocorrelation function of the strength function. The quantity (5.2) would be the survival probability of the system to remain at the initial state $|c\rangle$ after time $t$ if one uses the original strength function instead of $S^{(f)}$.

The properties of the square of the Fourier transform have recently been studied numerically as well as analytically in certain systems [6,19–23]. It was shown especially that there occurred a correlation hole at small $t$ when the system has a chaotic spectrum. For a regular spectrum, in contrast, the quantity has shown a monotonic decrease to its asymptotic value. In the present model system it is not *a priori* evident if a similar feature may be obtained. Here we are interested in the strength function of a state which couples to the background system, while previous works (on uncoupled systems) have studied, in a sense, the wave functions of the background system itself.

Figure 8 shows the square of the Fourier transform (smoothed over $\delta t = 0.1$) for the three values of $k$ and for the random matrix model. In the latter case an average over 50 choices of the realizations has been made. We notice that as the background system changes from regular to chaotic behavior the correlation hole becomes deeper in accordance with the findings of the previous works. According to Ref. [22], where a survival probability of a two-dimensional dynamical system was studied, its asymptotic value becomes smaller when the system becomes more chaotic. On the other hand, in our model where the parent state couples with a two-dimensional dynamical system, the asymptotic value of Eq. (5.2) becomes larger when the system becomes more chaotic. This contrasted result may be understood as follows: the inverse of the asymptotic value can be considered as an effective number of states to which the state $|c\rangle$ decays. In our model, this state couples with all the basis vectors of the background system with equal strengths. This feature does not change very much for regular systems even after the diagonalization of the background system as seen in the distribution of the coupling matrix elements $v_i$. The basis vectors of the background system, however, will be considerably mixed up for chaotic systems, causing a large fluctuation in the coupling. This will result in the reduction of the effective number of states with which our state $|c\rangle$ couples. Note that the behavior of asymptotic value depends on the choice of the coupling Hamiltonian.

A remark is in order. We did not see a correlation hole when we performed a Fourier transform of the original strength function instead of $S^{(f)}(E)$. This may be contrasted to the correlation hole that appeared when we performed the Fourier transform of the strength function with unfolded energies and constant strength.

## VI. SUMMARY

We adopted a model where a parent state couples with a large number of background states. We took as the background system a coupled two-dimensional anharmonic oscillator. The dynamics of this background system have been chosen classically integrable, irregular, or chaotic by changing a single parameter. We studied a strength function of the parent state, paying attention to its fluctuation properties, in order to find out whether differences of the dynamical properties of the background system could be reflected in the strength function.

First we studied some properties of the coupling matrix elements between the parent state and the background states. The distribution of the coupling matrix elements is different when the dynamics of the background is different.

We investigated several quantities reflecting a gross structure as well as different aspects of the fluctuation properties of the strength function. For a quantitative discussion of a gross structure of the strength function, we considered the energy moments of the strength function, and found that there are no differences. We then studied the distribution of the strength. When the background system is classically chaotic, the strength distribution follows a Gaussian similar to

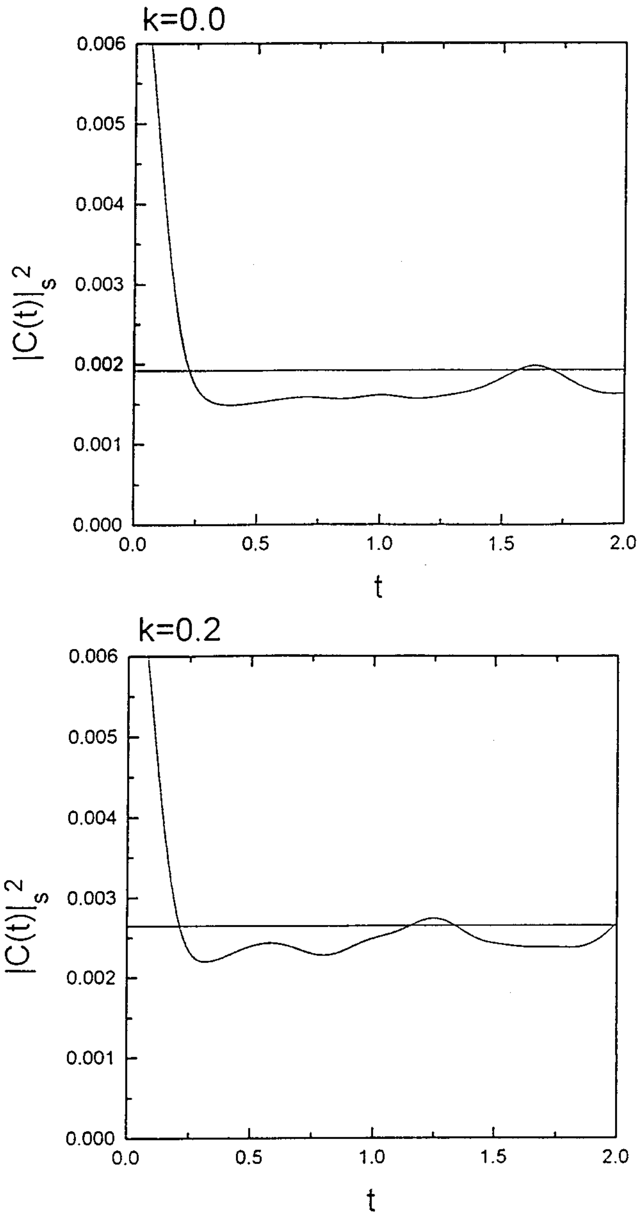


FIG. 8. Square of the Fourier transform smoothed over $\delta t = 0.1$ of the strength function with unfolded energy and modified strength as a function of time (measured in unit of $2\pi$) for three values of $k$ and also for the random matrix model (average of 50 cases). The horizontal lines denote asymptotic values which correspond to the first term of Eq. (5.2), and are 0.0019, 0.0026, 0.0032, and 0.0038 for the above four cases, respectively.

the case of the random matrix model. On the other hand, as the background system becomes more integrable or regular, it deviates more from a Gaussian. This result is directly attributed to the distribution of the coupling matrix elements. Next we performed a moment analysis of the strength function based on the partition function. When the energy resolution is larger than the mean level distance, the partition function reflects only the global energy variation of the coupling matrix elements thus showing no dependence on the parameter of the background system. On the other hand, by means of the same analysis we found the difference between the chaotic case and the case of the random matrix model. We also studied the Fourier transform of the autocorrelation function of the strength function. It is found that as the dynamical nature of the background system changes from integrable to chaotic, the correlation hole of the Fourier transform becomes deeper. The correlation hole is actually related to the spacing of unfolded levels. We also found that the asymptotic value of the Fourier transform becomes larger when the dynamics of the background becomes more chaotic. This is because when the parent state couples with every unperturbed background state with equal strength as in our model, the chaoticity of the background causes a large fluctuation in the coupling and reduces the effective number of states to which the parent state decays.

We stress that the above conclusions depend strongly on the coupling Hamiltonian of the parent state and the background system. In this paper we took one ansatz as these couplings. Other choices may give rise to different results for the fluctuation characteristics. For instance, if the parent state couples to only one unperturbed background state, the distribution of coupling matrix elements becomes equal to that of amplitudes of background states, leading to a different strength distribution from that of this paper.

For an application to realistic response phenomena such as giant resonances in nuclei, one must accordingly modify the model. The number of states which couple to the background system must be increased, and the collective state which is constructed as a superposition of these states should be considered. A study of such systems composed of many states coupling to a background system is now in progress.

## ACKNOWLEDGMENTS

The Joint Institute for Heavy Ion Research has as member institutions the University of Tennessee, Vanderbilt University, and the Oak Ridge National Laboratory; it is supported by the members and by the Department of Energy through Contract No. DE-FG05-87ER40361 with the University of Tennessee.

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